Radiatively Generated Neutrino Masses in $SU(3)_L \times U(1)_N$ Gauge Models

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In $SU(3)_L \times U(1)_N$ gauge models for electroweak interactions, we discuss how to implement a radiative mechanism of generating Majorana neutrino masses by considering the property that the Higgs scalar, which has a coupling to a charged lepton (ℓ)-neutrino (ν) pair, is naturally included. The mechanism is shown to work in models with a heavy charged lepton, ω^+, in a lepton triplet (ν, ℓ, ω^+) and with a heavy neutral lepton, N, in (ν, ℓ, N). A minimal model with ℓ and ℓ^c in (ν, ℓ, ℓ^c) together with a sextet Higgs scalar suffers from a fine-tuning problem to suppress tree-level neutrino masses.

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It is our common understanding that neutrinos are massive and oscillate among families [1]. The first experimental evidence is provided by the SuperKamiokande collaboration [2], which has observed atmospheric neutrino oscillations. There have been indications of solar neutrino oscillations [3], which are awaiting to be confirmed. Neutrinos produced at Los Alamos may also oscillate [4] and will be tested for their behavior at other laboratories [5]. Theoretically, neutrinos can acquire Majorana masses if the lepton number conservation is not respected. The origin of the lepton number violation is currently ascribed either to the formation of Majorana masses for right-handed neutrinos or to the existence of interactions involving a charged lepton-neutrino pair. The former is usually referred to as a seesaw mechanism [6] and the latter is referred to as a radiative mechanism [7]. Any viable models describing physics beyond the one based on the standard $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge model should contain mechanisms that accommodate massive neutrinos.

Among the possible extensions of the standard model, electroweak models based on an $SU(3)_L \times U(1)_N$ gauge group [8–17] provide the intriguing aspect that the models predict three families of quarks and leptons if the anomaly free condition on $SU(3)_L \times U(1)_N$ and the asymptotic free condition on $SU(3)_c$ are imposed. Another virtue of the models lies in the fact that the notion of the lepton number loses its meaning since a lepton triplet simultaneously contains the lepton and its antiparticle. Therefore, interactions that provide a coupling to a charged lepton-neutrino pair are expected to be naturally incorporated and can be utilized to radiatively generate neutrino masses [11,12]. The experimentally suggested magnitude of neutrino masses is at most of order 1 eV [18], which is much smaller than the mass of the electron. In the radiative mechanism, the lightness of neutrinos will be ensured by the smallness of the charged lepton masses and by weak couplings associated with lepton-number violating interactions.

In this report, we discuss how lighter masses of neutrinos are generated in $SU(3)_L \times U(1)_N$ gauge models. The minimal model contains left (L)-handed lepton states of the neutrino (ν_L) and the charged lepton (ℓ_L) in each family, which belong to the same triplet:

$$\psi_L^{i=1,2,3} = \left(\nu^i, \ell^i, \ell^{ci}\right)_L^T : (\mathbf{3}, 0), \tag{1}$$

where i = 1,2,3 stands for the family index, the superscript c of ℓ denotes the charge conjugation and the values in the parentheses specify quantum numbers based on the ($SU(3)_L$, $U(1)_N$) - symmetry. Let $N/2$ be the $U(1)_N$ quantum number. Then, the hypercharge, Y, is given by $Y = -\sqrt{3}\lambda^8 + N$ and the electric charge, Q_{em}, is given by $Q_{em} = (\lambda^3 + Y)/2$, where λ^a is the $SU(3)$ generator with $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$ ($a, b = 1\sim 8$). The quark sector is described by two quark antitriplets and one quark triplet, which are denoted by

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$$Q_L^{i=1,2} = \left(d^i, -u^i, J^i\right)_L^T : (\mathbf{3}^*, -\tfrac{1}{3}), \quad Q_L^3 = \left(u^3, d^3, J^3\right)_L^T : (\mathbf{3}, \tfrac{2}{3}), \tag{2}$$

where, for instance, Q_L^3 can be taken as quarks in the third family. It is obvious that the pure $SU(3)_L$ anomaly vanishes since there is an equal number of triplets of quarks and leptons and antitriplets of quarks. The right (R)-handed partners are classified as

$$u_R^{i=1,2,3} : (\mathbf{1}, \tfrac{2}{3}), \quad d_R^{i=1,2,3} : (\mathbf{1}, -\tfrac{1}{3}), \quad J_R^{i=1,2} : (\mathbf{1}, -\tfrac{4}{3}), \quad J_R^3 : (\mathbf{1}, \tfrac{5}{3}). \tag{3}$$

The Higgs scalars are used to break $SU(3)_L \times U(1)_N$ down to $U(1)_{em}$ via the standard $SU(2)_L \times U(1)_Y$ gauge group, where $U(1)_{em}$ stands for the electromagnetic $U(1)$ gauge group. The minimal set is supplied by

$$\eta = \left(\eta^0, \eta^-, \eta^+\right)^T : (\mathbf{3}, 0), \; \rho = \left(\rho^+, \rho^0, \rho^{++}\right)^T : (\mathbf{3}, 1), \; \chi = \left(\chi^-, \chi^{--}, \chi^0\right)^T : (\mathbf{3}, -1), \tag{4}$$

which will develop vacuum expectation values (VEV's):

$$\langle 0|\eta|0\rangle = (v_\eta, 0, 0)^T, \quad \langle 0|\rho|0\rangle = (0, v_\rho, 0)^T, \quad \langle 0|\chi|0\rangle = (0, 0, v_\chi)^T. \tag{5}$$

Masses of quarks and leptons are generated by Yukawa interactions:

$$\begin{aligned} -\mathcal{L}_Y = {} & \frac{1}{2}\epsilon^{\alpha\beta\gamma} G_{[ij]} \overline{\left(\psi_{\alpha L}^i\right)^c} \psi_{\beta L}^j \eta_\gamma + \sum_{i=1}^{2} \left(\overline{Q_L^i} D_R^i \eta^* + \overline{Q_L^i} U_R^i \rho^* + \overline{Q_L^i} C_R^i \chi^* \right) \\ & + \overline{Q_L^3} U_R^3 \eta + \overline{Q_L^3} D_R^3 \rho + G_{J3} \overline{Q_L^3} J_R^3 \chi + (\text{h.c.}) \end{aligned} \tag{6}$$

with $U_R^i = \sum_{j=1}^3 G_{uj}^i u_R^j$, $D_R^i = \sum_{j=1}^3 G_{dj}^i d_R^j$ and $C_R^i = \sum_{j=1}^2 G_{Jj}^i J_R^j$, where α, β, γ stand for the $SU(3)_L$ indices and G's ($G_{[ij]} = -G_{[ji]}$) denote the Yukawa couplings. The leptonic part contains lepton-number-violating interactions. Exotic quarks receive their masses through $\langle 0|\chi|0\rangle$. Because the lepton mass matrix turns out to be proportional to $G_{[ij]}$, which is antisymmetric with respect to the flavor index, it yields one massless lepton and two degenerated massive leptons, which are not phenomenologically accepted. To remedy this situation, one can consider three options [19]:

1. to introduce a sextet Higgs scalar to produce a flavor-symmetric mass matrix [20],
2. to replace ℓ_L^{ci} by a heavy charged lepton ω_L^{+i} with ω_R^{+i} and ℓ_R^i added [21] and
3. to replace ℓ_L^{ci} by the antineutrino ν_L^{ci} with ℓ_R^i added [9,11].

The rest of our discussions deals with how radiative generation of Majorana neutrino masses is implemented in these variants. In the minimal model, the mass term of $\ell_L^i \ell_L^{cj}$ is always accompanied by the lepton-number-violating term of $\nu_L^i \ell_L^j$, which is the main source for Majorana neutrino masses. This property is also inherited by the variants. The Higgs scalars exhibit various couplings, which will be constrained so as to incorporate a radiative mechanism for neutrino masses.

Let us first examine the model with a sextet Higgs scalar, $S : (\mathbf{6}^*, 0)$, which gives flavor-symmetric Yukawa interactions:

$$G_{\{ij\}} \overline{\left(\psi_{\alpha L}^i\right)^c} \psi_{\beta L}^j S^{\alpha\beta} + (\text{h.c.}), \tag{7}$$

where $G_{\{ij\}} = G_{\{ji\}}$. The S^{11} scalar, σ_1^0, has a coupling to the neutrino. It has been stressed in Ref. [20] that σ_1^0 will readily develop a VEV through other Higgs interactions. As a result, neutrinos get Majorana masses at the tree level, which should be forbidden since we are interested in radiative generation of neutrino masses. Furthermore, since σ_1^0 belongs to a triplet of the standard $SU(2)_L$ gauge group, phenomenology of weak interactions also restricts $\langle 0|\sigma_1^0|0\rangle$ to be much smaller than the VEV of the $SU(2)_L$-doublet Higgs scalar. One may have a fine-tuning among various Higgs couplings to yield vanishing neutrino masses at the tree level and a radiative mechanism is, then, operative [22].

We impose an extra symmetry that permits maintaining $\langle 0|\sigma_1^0|0\rangle = 0$ regardless of any Higgs interactions. [1] The discrete symmetry given by $\eta \to -\eta$, $\rho \to i\rho$, $\chi \to i\chi$, $S \to -S$, $\psi_L \to i\psi_L$, $Q_L^{i=1,2} \to -iQ_L^i$, $Q_L^3 \to -Q_L^3$, $u_R^i \to$

[1] There was a suggestion that the condition of $\langle 0|\sigma_1^0|0\rangle = 0$ could be linked to the masslessness of the photon, which, however, may not be the case. See Ref. [13] for the details.

u_R^i, $d_R^i \to i d_R^i$, $J_R^{i=1,2} \to J_R^i$ and $J_R^3 \to i J_R^3$ [20] forbids dangerous couplings such as $\eta S \eta$, det$S$, $\epsilon_{\alpha\beta\gamma} (S\eta)^\alpha \rho^{c\beta}\chi^{c\gamma}$, $\epsilon_{\alpha\beta\gamma}\epsilon_{\alpha'\beta'\gamma'}S^{\alpha\alpha'}S^{\beta\beta'}S^{\gamma\gamma'}$ and $\epsilon_{\alpha\beta\gamma}\epsilon_{\alpha'\beta'\gamma'}S^{\alpha\beta}S^{\alpha'\beta'}\rho^{c\gamma}\chi^{c\gamma'}$, which tend to align $\langle 0|\sigma_1^0|0\rangle \neq 0$ [23]. Instead, the required coupling to align $\langle 0|\sigma_1^0|0\rangle = 0$ is represented by $\rho S \chi$, which is allowed. Since the Majorana neutrino mass term behaves as $\sigma_1^{0\dagger}$, the neutrino (ν) will have an effectively generated $\nu\sigma_1^0\nu$-coupling once it acquires a mass. This shows that any attempts to generate neutrino masses rely upon $\langle 0|\sigma_1^0|0\rangle \neq 0$. In other words, to generate Majorana neutrino masses, one needs a Higgs interaction that is not respected by the discrete symmetry. The Higgs potential including such an interaction will necessarily yield $\langle 0|\sigma_1^0|0\rangle \neq 0$. Therefore, the sextet Higgs scalar should not be employed.

Next, we examine the second model with heavy charged leptons, which are introduced in each family. The lepton sector is modified to:

$$\psi_L^i = \left(\nu^i, \ell^i, \omega^i\right)_L^T : (\mathbf{3}, 0), \quad \ell_R^i : \ (\mathbf{1}, -1), \quad \omega_R^i : \ (\mathbf{1}, 1), \tag{8}$$

where we have denoted ω^{+i} by ω^i. Their Yukawa interactions are given by

$$\overline{\psi_L^i}\left(L^i\rho + \Omega^i\chi\right) + (\text{h.c}) \tag{9}$$

with the definitions of $L^i = \sum_{j=1}^3 G_{\ell j}^i \ell_R^j$ and $\Omega^i = \sum_{j=1}^3 G_{\omega j}^i \omega_R^j$. In this case, the neutrino mass term behaves as $\eta^{0\dagger}\eta^{0\dagger}$ instead of $\sigma_1^{0\dagger}$. The possible Higgs coupling,

$$\lambda\left(\rho^\dagger\eta\right)\left(\chi^\dagger\eta\right) + (\text{h.c.}), \tag{10}$$

is chosen to be responsible for generating Majorana neutrino masses, where λ is a coupling strength. The most general Higgs potential is, thus, described by self-Hermitian terms composed of $\phi_\alpha\phi_\beta^\dagger$ ($\phi = \eta,\ \rho,\ \chi$) and by the non-self-Hermitian term of $(\rho^\dagger\eta)(\chi^\dagger\eta)$ as well as by the familiar term of $\epsilon^{\alpha\beta\gamma}\eta_\alpha\rho_\beta\chi_\gamma$ [8].

The relevant Yukawa interactions, which retain the $\psi\psi\eta$ - part in Eq.(6) as a lepton-number-violating source, are given by

$$\begin{aligned} &-\frac{1}{2}G_{[ij]}\left[\left(\overline{\ell_R^{ci}}\nu_L^j - \overline{\nu_R^{ci}}\ell_L^j\right)\eta^+ + \left(\overline{\omega_R^{ci}}\ell_L^j - \overline{\ell_R^{ci}}\omega_L^j\right)\eta^0 + \left(\overline{\nu_R^{ci}}\omega_L^j - \overline{\omega_R^{ci}}\nu_L^j\right)\eta^-\right] \\ &\qquad -G_{\ell i}^i\left(\overline{\nu_L^i}\ell_R^i\rho^+ + \overline{\ell_L^i}\ell_R^i\rho^0\right) - G_{\omega i}^i\left(\overline{\nu_L^i}\omega_R^i\chi^- + \overline{\omega_L^i}\omega_R^i\chi^0\right) + (\text{h.c.}), \end{aligned} \tag{11}$$

where the charged lepton and heavy lepton mass matrices are assumed to be diagonal for simplicity. The one-loop diagrams of the Zee's radiative type [7] contain three processes via charged leptons, via heavy charged leptons and via their mixings, which are shown in Fig.1(a) $\sim$ (c). These diagrams correspond to an effective coupling of

$$\left(\eta^\dagger\psi_L\right)\epsilon^{\alpha\beta\gamma}\rho_\alpha\chi_\beta\psi_{\gamma L} \tag{12}$$

as a Majorana mass term [24]. The resulting neutrino mass matrix turns out to take the following form:

$$\begin{pmatrix} \delta_{11} & m_{12}+\delta_{12} & m_{13}+\delta_{13} \\ m_{12}+\delta_{12} & \delta_{22} & m_{23}+\delta_{23} \\ m_{13}+\delta_{13} & m_{23}+\delta_{23} & \delta_{33} \end{pmatrix}, \tag{13}$$

where m_{ij} and δ_{ij} are calculated to be

$$m_{ij} = \lambda G_{[ij]}\left[\frac{m_{\ell_j}^2 - m_{\ell_i}^2}{v_\rho^2}F(m_{\eta^+}^2, m_{\rho^+}^2) + \frac{m_{\omega_j}^2 - m_{\omega_i}^2}{v_\chi^2}F(m_{\eta^-}^2, m_{\chi^-}^2)\right]v_\eta v_\rho v_\chi \quad (i \neq j), \tag{14}$$

$$\delta_{ij} = \frac{\lambda}{4}\sum_{k,\ell}G_{[ik]}G_{[k\ell]}G_{[\ell j]}\left[G(m_{\omega_k}^2, m_\eta^2) - G(m_{\omega_\ell}^2, m_\eta^2)\right]v_\eta v_\rho v_\chi \quad (i \neq j), \tag{15}$$

$$\delta_{ii} = \frac{\lambda}{4}G_{[12]}G_{[23]}G_{[31]}\sum_{j,k}\epsilon_{ijk}\left[G(m_{\omega_j}^2, m_\eta^2) - G(m_{\omega_k}^2, m_\eta^2)\right]v_\eta v_\rho v_\chi, \tag{16}$$

and

$$F(x,y) = \frac{1}{16\pi^2}\frac{\log x - \log y}{x-y}, \quad G(x,y) = \frac{1}{16\pi^2}\frac{1}{x-y}\left[x\frac{\log x - \log y}{x-y} - 1\right]. \tag{17}$$

As mass parameters, $m_{\eta^\pm,\rho^+,\chi^-}$ are the masses of Higgs scalars, $\eta^\pm$, ρ^+ and χ^-, and m_{ℓ_i} ($\equiv G^i_{\ell i}v_\rho$) and m_{ω_i} ($\equiv G^i_{\omega i}v_\chi$) are, respectively, the mass of the i-th charged lepton and the mass of the i-th heavy charged lepton. The explicit form of $G(x,y)$ is subject to the condition of $m_{\eta^+} = m_{\eta^-}$ ($=m_\eta$) with m_{ℓ_i} neglected. We are anticipating that the mass, δ_{ij}, is extremely small since it is proportional to the cubic of the lepton-number-violating coupling of $G_{[ij]}$.

Expected neutrino phenomenology arising from the mass matrix of (13) has been extensively discussed in Ref. [25]. For example, to get bi-maximal neutrino mixing, one has to require that $m_{12} \sim m_{13} \sim 0.01$ eV together with $m_{13}/m_{23} \sim \Delta m^2_{atm}/\Delta m^2_\odot$, where $\Delta m^2_{atm} \sim 10^{-3}$ eV^2 and $\Delta m^2_\odot \sim 10^{-5}$ or 10^{-10} eV^2. To figure out the order of magnitudes for $G_{[ij]}$, let us specify various mass parameters. For values of $v_{\eta,\rho,\chi}$, $v_{\eta,\rho}$ yield masses of weak bosons and v_χ is a source of masses for heavy charged leptons, exotic quarks and exotic gauge bosons. So, let us use (v_η, v_ρ, v_χ) $\sim$ (100, 100, 1000) GeV as typical values, $m_{\omega_i} \sim 100$ GeV as masses for the heavy charged leptons and $\mathcal{O}(300)$ GeV as masses for the mediating charged Higgs scalars. We find that, for Δm_{ij} representing $|m_{\omega_i} - m_{\omega_j}|$, $m_{ij} \sim G_{[ij]}\lambda(\Delta m_{ij}/1\mathrm{GeV}) \times 10^{-5}$ (GeV) for the heavy-lepton-exchange. For $\Delta m_{ij} \sim 10$ GeV, where the heavy-lepton-exchange dominates the charged-lepton-exchange, we obtain $G_{[12]}\lambda \sim G_{[13]}\lambda \sim 10^{-7}$ with $G_{[23]}/G_{[13]} \sim 10^{-2}$ or 10^{-7}.

The last example to avoid flavor-antisymmetric mass terms is to put ν^i and ν^{ci} together in a lepton triplet, which is described by

$$\psi^i_L = \left(\nu^i, \ell^i, \nu^{ci}\right)^T_L : (\mathbf{3}, -\tfrac{1}{3}), \quad \ell^i_R : (\mathbf{1}, -1), \tag{18}$$

where $Y = -\lambda^8/\sqrt{3} + N$. The quark sector consists of two antitriplets with extra heavy down quarks, d', and one triplet with an extra heavy up quark, u':

$$\begin{aligned} &Q^{i=1,2}_L = \left(d^i, -u^i, d'^i\right)^T_L : (\mathbf{3}^*, 0), \quad Q^3_L = \left(u^3, d^3, u'^3\right)^T_L : (\mathbf{3}, \tfrac{1}{3}), \\ &u^{1,2,3}_R : (\mathbf{1}, \tfrac{2}{3}), \quad d^{1,2,3}_R : (\mathbf{1}, -\tfrac{1}{3}), \quad d'^{1,2}_R : (\mathbf{1}, -\tfrac{1}{3}), \quad u'^3_R : (\mathbf{1}, \tfrac{2}{3}). \end{aligned} \tag{19}$$

As discussed in Ref. [11], it is possible to radiatively generate neutrino masses of the Dirac type, especially for the electron neutrino, by introducing four Higgs scalars:

$$\begin{aligned} &\rho = (\rho^+, \rho^0, \bar\rho^+)^T : (\mathbf{3}, \tfrac{2}{3}), \quad \rho' = (\rho'^+, \rho'^0, \bar\rho'^+)^T : (\mathbf{3}, \tfrac{2}{3}), \\ &\eta = (\eta^0, \eta^-, \bar\eta^0)^T : (\mathbf{3}, -\tfrac{1}{3}), \quad \chi = (\bar\chi^0, \chi^-, \chi^0)^T : (\mathbf{3}, -\tfrac{1}{3}), \end{aligned} \tag{20}$$

and other sextets. To avoid unwanted flavor changing effects is done by the orthogonal choice of $\langle 0|\eta|0\rangle = (v_\eta,\ 0,\ 0)^T$ and $\langle 0|\chi|0\rangle = (0,\ 0,\ v_\chi)^T$, which is ensured by the appropriate Higgs potential [19,20,26]. Masses of quarks and leptons are supplied by $\langle 0|\rho^0|0\rangle$, but the extra Higgs scalar, ρ', is requested to develop no VEV, *i.e.* $\langle 0|\rho'^0|0\rangle = 0$.

By employing the same Higgs scalars but without sextets, one can argue that neutrinos acquire radiatively generated Majorana masses. Since ν^{ci}_L remains massless, its right handed partner, ν^{ci}_R, is introduced to form a mass term with ν^{ci}_L. [2] It is the model with heavy neutral leptons (N^i) [27], where the lepton multiplets in Eq.(18) are modified to

$$\psi^i_L = \left(\nu^i, \ell^i, N^i\right)^T_L : (\mathbf{3}, -\tfrac{1}{3}), \quad \ell^i_R : (\mathbf{1}, -1), \quad N^i_R : (\mathbf{1}, 0). \tag{21}$$

The Yukawa and Higgs interactions are subject to the following discrete symmetry based on a Z_2 parity that distinguishes between ρ and ρ' as well as between η and χ. The Z_2 parity is negative for (ψ^i_L, $Q^{1,2}_L$, $d^{1,2,3}_R$, u'^3_R, ρ, χ) and positive for all others.

The relevant Higgs coupling, which are responsible for neutrino masses, are specified by

$$\lambda_1 \left(\rho^\dagger\eta\right)\left(\chi^\dagger\rho'\right) + \lambda_2 \left(\rho^\dagger\rho'\right)\left(\chi^\dagger\eta\right) + (\text{h.c.}), \tag{22}$$

and the Yukawa couplings are given by

$$\begin{aligned} -\mathcal{L}_Y = &\frac{1}{2}\epsilon^{\alpha\beta\gamma}G_{[ij]}\overline{\left(\psi^i_{\alpha L}\right)^c}\psi^j_{\beta L}\rho'_\gamma + G^i_{\ell i}\overline{\psi^i_L}\ell^i_R\rho + G^i_{Ni}\overline{\psi^i_L}N^i_R\chi, \\ &+ \sum_{i=1}^{2}\left(\overline{Q^i_L}D^i_R\eta^* + \overline{Q^i_L}U^i_R\rho^* + \overline{Q^i_L}C^i_R\chi^*\right) + \overline{Q^3_L}U^3_R\eta \\ &+ \overline{Q^3_L}D^3_R\rho + G_{u'3}\overline{Q^3_L}u'^3_R\chi + (\text{h.c.}), \end{aligned} \tag{23}$$

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[2] Instead of introducing ν^{ci}_R, one can use the sextet to develop a Majorana mass term of $\nu^{ci}_L\nu^{ci}_L$ by $\langle 0|S^{33}|0\rangle$ as in Eq.(7). However, it needs careful setup for the sextet to develop no Majorana mass of $\nu^i_L\nu^i_L$ by $\langle 0|S^{11}|0\rangle$.

with U^i_R and D^i_R are the same as in Eq.(6) and $C^i_R = \sum_{j=1}^2 G'^i_{d'j} d'^j_R$. The charged and neutral lepton mass matrices have been set to be diagonal. The requirement of $\langle 0|\rho'^0|0\rangle = 0$ should be always maintained to keep neutrinos massless at the tree level and is fulfilled as a result of the discrete symmetry. The corresponding one-loop diagrams are given by Fig.1(a) with $\eta^+ \to \bar{\rho}'^+$ and by Fig.1(b) with $(\chi^-, \eta^-, \omega^i) \to (\bar{\chi}^0, \rho'^0, N^i)$. The resulting neutrino mass matrix is reproduced by Eq.(13) with δ_{ij} =0 and evaluated neutrino masses are given by Eq.(14), where the above replacement is performed and the coefficient λ is replaced by λ_1 (λ_2) for the charged (heavy)-lepton-exchange. The similar values of $G_{[ij]}$ to the previous ones are, thus, obtained.

The discrete symmetry forbids the Yukawa couplings: $\epsilon^{\alpha\beta\gamma}\psi^i_{\alpha L}\psi^j_{\beta L}\rho_\gamma$ and $\overline{\psi^i_L}\ell^i_R\rho'$, the former of which would yield a mixing term of $\nu^i_L N^j_L$ at the tree level. Also forbidden are dangerous Higgs couplings: $\epsilon^{\alpha\beta\gamma}\eta_\alpha\rho'_\beta\chi_\gamma$ and $\rho^\dagger\rho'$ that would align $\langle 0|\rho'^0|0\rangle \neq 0$ and $\eta^\dagger\chi$ that would disturb the orthogonal choice of $\langle 0|\eta|0\rangle$ and $\langle 0|\chi|0\rangle$. On the other hand, $\epsilon^{\alpha\beta\gamma}\eta_\alpha\rho_\beta\chi_\gamma$ is allowed and gives $(\eta^0\chi^0$ - $\bar{\eta}^0\bar{\chi}^0)\rho^0$ in the Higgs potential. The VEV's of $\langle 0|\eta^0|0\rangle$ and $\langle 0|\chi^0|0\rangle$ are preferred by $\eta^0\chi^0\rho^0$ giving a negative contribution to the Higgs potential, which corresponds to a positive contribution of $\bar{\eta}^0\bar{\chi}^0\rho^0$. Therefore, $\epsilon^{\alpha\beta\gamma}\eta_\alpha\rho_\beta\chi_\gamma$ helps to align $\langle 0|\bar{\eta}^0|0\rangle = 0$ and $\langle 0|\bar{\chi}^0|0\rangle = 0$. The non-self Hermitian part of the Higgs potential includes $\epsilon^{\alpha\beta\gamma}\eta_\alpha\rho_\beta\chi_\gamma$, $(\chi^\dagger\eta)^2$, $(\rho^\dagger\rho')^2$ and Eq.(22). Finally, since the $SU(2)_L$-singlet exotic quarks have the same charges as the $SU(2)_L$-doublet ordinary quarks, there are dangerous flavor-changing Higgs interactions [28]. For example, they can be suppressed by restricting the Yukawa interactions for quarks by imposing the discrete symmetry. The corresponding Z_2 parity is negative for (ψ^i_L, $u^{1,2}_R$, u'^3_R, d^3_R, $d'^{1,2}_R$, ρ, χ) [14] instead of the previous one: (ψ^i_L, $Q^{1,2}_L$, $d^{1,2,3}_R$, u'^3_R, ρ, χ).

It should be noted that, in the present context with the assignment of (ν^i, ℓ^i, ν^{ci}) as in Eq.(18), Dirac neutrino masses would be generated if there were the transition of $\bar{\rho}'^+ \leftrightarrow \bar{\rho}^+$. It is possible by the Higgs couplings of $\left(\rho^\dagger\chi\right)\left(\chi^\dagger\rho'\right)$, which would not disturb $\langle 0|\rho'^0|0\rangle = 0$. However, if this coupling is allowed, one cannot forbid $\rho^\dagger\rho'$ by any symmetries, which necessarily causes $\langle 0|\rho'^0|0\rangle \neq 0$. Then, Dirac neutrino masses are present at the tree level.

In summary, we have succeeded in accommodating a radiative mechanism for Majorana neutrino masses in models based on $SU(3)_L \times U(1)_N$. Heavy leptons are required for the mechanism to consistently work. In the model with heavy neutral leptons, it is essential to employ one "silent" Higgs scalar that is nothing to do with the symmetry breakdown. On the other hand, the model including heavy charged leptons needs no extra Higgs scalars. All Higgs scalars are just contained in a minimal set, which gives a correct symmetry breaking of $SU(3)_L \times U(1)_N$ as well as masses of quarks and leptons. The essence lies in the property that a Higgs triplet necessarily includes a charged Higgs scalar that couples to a charged lepton-neutrino pair, which plays an important role in the radiative mechanism. We hope that one of the models based on $SU(3)_L \times U(1)_N$ serves as the promising model beyond the standard model as far as radiative generation of neutrino masses is concerned.

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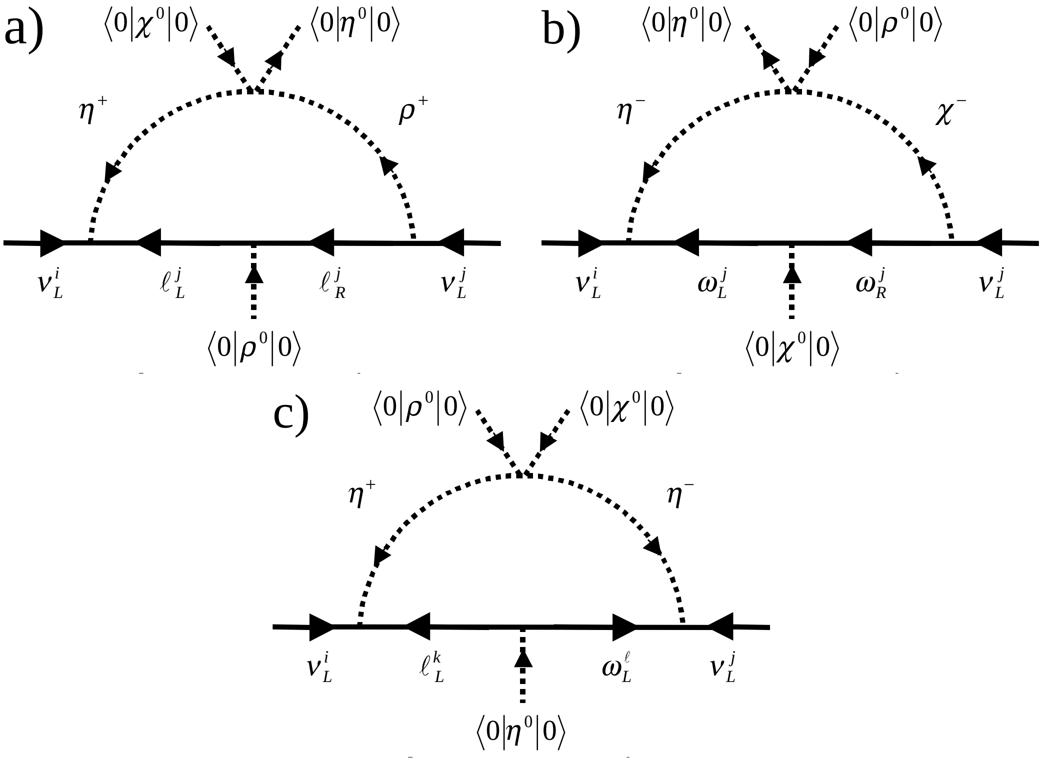
Figure Caption

Fig.1 : One loop radiative diagrams for ν^i-ν^j via (a) charged leptons and (b) heavy leptons for $i \neq j$ and via (c) the mixing between charge leptons and heavy leptons for $i \neq k$, $k \neq \ell$ and $\ell \neq j$.



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